# ITRF2014: Equations of post-seismic deformation models

After an Earthquake, the position of a station during the post-seismic trajectory, $X_{PSD}$, at an epoch $t$ could be written as:

$$X_{PSD}(t) = X(t_0) + \dot{X}(t - t_0) + \delta X_{PSD}(t) \tag{1}$$

where $\dot{X}$ is the station linear velocity vector, and $\delta X_{PSD}(t)$ is the total sum of the post-seismic deformation (PSD) corrections at epoch $t$. For each component $L \in$ {E,N,U}, we note $\delta L$ the total sum of PSD corrections expressed in the local frame at epoch $t$:

$$\delta L(t) = \sum_{i=1}^{n^l} A_i^l \log(1 + \frac{t - t_i^l}{\tau_i^l}) + \sum_{i=1}^{n^e} A_i^e (1 - \mathrm{e}^{-\frac{t - t_i^e}{\tau_i^e}}) \tag{2}$$

where:

$n^l$ : Number of logarithmic terms of the parametric model
$n^e$ : Number of exponential terms of the parametric model
$A_i^l$ : Amplitude of the $i^{th}$ logarithmic term
$A_i^e$ : Amplitude of the $i^{th}$ exponential term
$\tau_i^l$: Relaxation time of the $i^{th}$ logarithmic term
$\tau_i^e$: Relaxation time of the $i^{th}$ exponential term
$t_i^l$: Earthquake time(date) corresponding to $i^{th}$ logarithmic term
$t_i^e$: Earthquake time(date) corresponding to the $i^{th}$ exponential term

The variance of $\delta L(t)$ is given by:

$$\mathrm{var}(\delta L) = C.\mathrm{var}(\theta).C^T \tag{3}$$

where $\theta$ is the vector of parameters of the post-seismic deformation model:

$$\theta = [A_1^l, \tau_1^l, ...., A_{n^l}^l, \tau_{n^l}^l, A_1^e, \tau_1^e, ...., A_{n^e}^e, \tau_{n^e}^e]$$

The elements of the matrix $C$ are computed by the following formulas:

$$\frac{\partial \delta L}{\partial A_i^l} = \log(1 + \frac{t - t_i^l}{\tau_i^l}) \tag{4}$$

$$\frac{\partial \delta L}{\partial \tau_i^l} = -\frac{A_i^l (t - t_i^l)}{(\tau_i^l)^2 (1 + \frac{t - t_i^l}{\tau_i^l})} \tag{5}$$

$$\frac{\partial \delta L}{\partial A_i^e} = 1 - \mathrm{e}^{-\frac{(t - t_i^e)}{\tau_i^e}} \tag{6}$$

$$\frac{\partial \delta L}{\partial \tau_i^e} = -\frac{A_i^e (t - t_i^e) \mathrm{e}^{-\frac{(t - t_i^e)}{\tau_i^e}}}{(\tau_i^e)^2} \tag{7}$$

Note that the PSD models are determined and provided to the users per component $L \in$ {E,N,U}, independently, and so there are NO cross-terms (or correlations) between components. However, cross-terms between amplitude and relaxation time for each LOG or/and EXP term should be taken into account in the variance calculation of equation (3). As an example, if for a given station there are 3 earthquakes that were taken into account in the estimation of the PSD models of its component $E$, and it has one EXP for the first EQ, one EXP for the 2nd EQ and LOG+EXP for the 3rd EQ, the one line matrix $C$ for component $E$ in equation (3) will have 8 terms.

Once the variances $\mathrm{var}(\delta E), \mathrm{var}(\delta N), \mathrm{var}(\delta U)$ are computed, the transformation into cartesian is obtained by:

$$\mathrm{var}\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} = R\,.\begin{bmatrix} \mathrm{var}(\delta E) & 0 & 0 \\ 0 & \mathrm{var}(\delta N) & 0 \\ 0 & 0 & \mathrm{var}(\delta U) \end{bmatrix}.\,R^T \tag{8}$$

where R is the transformation (Jacobian) matrix from topocentric to geocentric frame, and where:

$$\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} = R\,.\begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix} \tag{9}$$