

## ITRF2020P: Equations of post-seismic deformation models

After an Earthquake, the position of a station during the post-seismic trajectory,  $X_{PSD}$ , at an epoch  $t$  could be written as:

$$X_{PSD}(t) = X(t_0) + \dot{X}(t - t_0) + \delta X_{PSD}(t) \quad (1)$$

where  $\dot{X}$  is the station linear velocity vector, and  $\delta X_{PSD}(t)$  is the total sum of the post-seismic deformation (PSD) corrections at epoch  $t$ . For each component  $L \in \{E, N, U\}$ , we note  $\delta L$  the total sum of PSD corrections expressed in the local frame at epoch  $t$ :

$$\delta L(t) = \sum_{i=1}^{n^l} A_i^l \log\left(1 + \frac{t - t_i^l}{\tau_i^l}\right) + \sum_{i=1}^{n^e} A_i^e \left(1 - e^{-\frac{t - t_i^e}{\tau_i^e}}\right) \quad (2)$$

where:

$n^l$  : Number of logarithmic terms of the parametric model

$n^e$  : Number of exponential terms of the parametric model

$A_i^l$  : Amplitude of the  $i^{th}$  logarithmic term

$A_i^e$  : Amplitude of the  $i^{th}$  exponential term

$\tau_i^l$  : Relaxation time of the  $i^{th}$  logarithmic term

$\tau_i^e$  : Relaxation time of the  $i^{th}$  exponential term

$t_i^l$  : Earthquake time(date) corresponding to  $i^{th}$  logarithmic term

$t_i^e$  : Earthquake time(date) corresponding to the  $i^{th}$  exponential term

The variance of  $\delta L(t)$  is given by:

$$\text{var}(\delta L) = C \cdot \text{var}(\theta) \cdot C^T \quad (3)$$

where  $\theta$  is the vector of parameters of the post-seismic deformation model:

$$\theta = [A_1^l, \tau_1^l, \dots, A_{n^l}^l, \tau_{n^l}^l, A_1^e, \tau_1^e, \dots, A_{n^e}^e, \tau_{n^e}^e]$$

The elements of the matrix  $C$  are computed by the following formulas:

$$\frac{\partial \delta L}{\partial A_i^l} = \log\left(1 + \frac{t - t_i^l}{\tau_i^l}\right) \quad (4)$$

$$\frac{\partial \delta L}{\partial \tau_i^l} = -\frac{A_i^l(t - t_i^l)}{(\tau_i^l)^2 \left(1 + \frac{t - t_i^l}{\tau_i^l}\right)} \quad (5)$$

$$\frac{\partial \delta L}{\partial A_i^e} = 1 - e^{-\frac{(t - t_i^e)}{\tau_i^e}} \quad (6)$$

$$\frac{\partial \delta L}{\partial \tau_i^e} = -\frac{A_i^e(t - t_i^e)e^{-\frac{(t - t_i^e)}{\tau_i^e}}}{(\tau_i^e)^2} \quad (7)$$

Note that the PSD models are determined and provided to the users per component  $L \in \{E, N, U\}$ , independently, and so there are NO cross-terms (or correlations) between components. However, cross-terms between amplitude and relaxation time for each LOG or/and EXP term should be taken into account in the variance calculation of equation (3). As an example, if for a given station there are 3 earthquakes that were taken into account in the estimation of the PSD models of its component  $E$ , and it has one EXP for the first EQ, one EXP for the 2nd EQ and LOG+EXP for the 3rd EQ, the one line matrix  $C$  for component  $E$  in equation (3) will have 8 terms.

Once the variances  $\text{var}(\delta E)$ ,  $\text{var}(\delta N)$ ,  $\text{var}(\delta U)$  are computed, the transformation into cartesian is obtained by:

$$\text{var} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} = R \cdot \begin{bmatrix} \text{var}(\delta E) & 0 & 0 \\ 0 & \text{var}(\delta N) & 0 \\ 0 & 0 & \text{var}(\delta U) \end{bmatrix} \cdot R^T \quad (8)$$

where  $R$  is the transformation (Jacobian) matrix from topocentric to geocentric frame, and where:

$$\begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} = R \cdot \begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix} \quad (9)$$